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Ionospheric Reflection of Small–Scale Alfvén Waves

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Abstract.
The transfer of energy from the magnetosphere to the ionosphere occurs through complex mechanisms that often involve shear Alfvén waves, implying that the interaction of these waves with the ionosphere is fundamental to our understanding of the coupling process. Indeed, a variety of problems that involve magnetosphere–ionosphere coupling assume that the ionosphere acts as a perfectly reflecting boundary, although we know that this is not the case in general. In this work, we calculate the reflection coefficient of shear Alfvén waves incident on the ionosphere under three different ionospheric profiles. Our results indicate that 1) under all profiles considered, reflection of shear Alfvén waves with perpendicular lengths of $\sim$ 1 km or less is negligible and 2) under a profile that might be typical of a nightside ionosphere, reflection of shear Alfvén waves is significant only for waves with frequencies below $\sim$ 0.4 Hz and λ_x above $\sim$ 2 km; reflection of waves with λ_x less than $\sim$ 2 km is poor.

1. Introduction

The reflection of small–scale shear Alfvén waves from the ionosphere is a problem with important implications in magnetosphere–ionosphere coupling. Indeed, mechanisms responsible for energy transfer from the magnetosphere to the ionosphere very often involve Alfvén waves, e.g auroral particle acceleration, cavity and waveguide modes, flux transfer events, the Alfvén resonator, etc. In many of these cases, the ionosphere is assumed to be a perfectly reflecting boundary for Alfvén waves. However, *Farley* [1959] showed that in order for perpendicular electrostatic fields to map through the ionosphere they must have perpendicular length scales greater than $\sqrt{\sigma_P/\sigma_0 L_d}$, where L_d is the extent in altitude of the dissipative region of the ionosphere. This important statement means that waves with small perpendicular wavelengths will tend to be absorbed by the ionosphere (and not reflected), with the degree of absorption depending on the ratio σ_P/σ_0.

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Further work related to reflections of low–frequency Alfvén waves from the ionosphere approximated the reflection coefficient τ by $(\Sigma_p^{-1} - Z_A)/(\Sigma_p^{-1} + Z_A)$, where Σ_p is the height–integrated conductivity of the ionosphere and Z_A is the Alfvén wave impedance $\mu_0 V_A$ above the ionosphere [*Scholer*, 1970; *Hughes*, 1974; *Vogt and Haerendel*, 1998]. *Hughes* [1974] determined that, for large–scale, low–frequency waves, the dayside ionosphere can be almost perfectly reflecting and later extended this work to include effects of different ionospheric profiles, showing that ionospheres typical of what might be found on the nightside may be poor reflectors [*Hughes and Southwood*, 1976].

For wave periods the order of a few seconds or less, the situation is different because the parallel wavelength becomes comparable to the vertical extent of the Earth–ionosphere cavity, which can then trap these waves in a manner similar to a waveguide [*Greifinger*, 1972]. Trapping of waves in the range of 0.1–1.0 Hz may also be possible in the region between the ionosphere and a peak in the Alfvén speed near $1R_E$ (the "Alfvén resonator") [*Lysak*, 1999]. In the model used to study characteristics that might be observed as result of wave propagation through this region, *Lysak* [1999] consider realistic ionospheric profiles, but used a perpendicular scale length of 250 km, so effects of short perpendicular wavelength are masked.

The goal of this work is to examine ionospheric reflection of shear Alfvén waves using realistic density profiles, with the objective of determining the effect on the reflection coefficient of short perpendicular wavelengths. The model used is based on that used by *Knudsen et al.* [1992], where the vertical profile of magnetic and electric fields of an Alfvén wave reflecting from the ionosphere were calculated.

2. Model description

The work presented here is an extension of that presented by *Knudsen et al.* [1992]. As in their work, this model solves Maxwell's equations, providing a full–wave solution to the problem of ionospheric reflection. Plasma effects are incorporated via a complex conductivity tensor.

We begin with Faraday's Law, $\nabla \times \mathbf{E} = -i\omega\mathbf{B}$ and Ampére's Law, $\nabla \times \mathbf{B} = \mu_0 \overleftrightarrow{\sigma} \cdot \mathbf{E} + \frac{i\omega}{c^2}\mathbf{E}$. The conductivity is derived from linearized fluid equations of motion as shown in *Knudsen et al.* [1992]. The resulting conductivity tensor includes Alfvén wave behavior for finite ω as well as the Pedersen, Hall and direct conductivities in the zero–frequency limit.

$$\overleftrightarrow{\sigma} = \begin{pmatrix} \tilde{\sigma}_1 & \tilde{\sigma}_2 & 0 \\ -\tilde{\sigma}_2 & \tilde{\sigma}_1 & 0 \\ 0 & 0 & \tilde{\sigma}_0 \end{pmatrix} \quad (1)$$

where

$$\tilde{\sigma}_0 = \varepsilon_0 \sum_j \frac{\omega_{pj}^2}{(i\omega + \nu_j)} \quad (2)$$

$$\tilde{\sigma}_1 = \varepsilon_0 \sum_j \frac{(i\omega + \nu_j)\,\omega_{pj}^2}{\left[(i\omega + \nu_j)^2 + \Omega_j^2\right]} \quad (3)$$

$$\tilde{\sigma}_2 = \varepsilon_0 \sum_j \frac{\Omega_j \omega_{pj}^2}{\left[(i\omega + \nu_j)^2 + \Omega_j^2\right]} \quad (4)$$

In the above equations, ω_{pj} is the plasma frequency for species j, ν_j is the effective collision frequency of the jth species, and the cyclotron frequency Ω_j has the same sign as the charge on j.

We assume that Earth's surface is flat and perfectly conducting and consider a wave that is manifested as periodic variations of the fields in the $\hat{x}$ (N-S) direction, and with no variations in the $\hat{y}$ (E-W) direction ($\hat{z}$ is upward). In addition, we assume that $\mathbf{B}_0$ is vertical and that $\overleftrightarrow{\sigma}$ is homogeneous in x and y and neglect coupling between fast mode and shear waves, which is small for short perpendicular wavelengths. Omission of E_y from the calculation assumes that the compressional mode carries much less energy than the shear mode, which is justified by Figure 7 of *Knudsen et al.* [1992], showing that $|E_y/E_x|$ is approximately 20% for λ_x =1000 km. While this magnitude of E_y is significant in the E region, the associated energy is only 4% of that of the shear mode, and has little effect on the overall wave Poynting flux or reflection properties. A full fourth-order solution shows that the ratio E_y/E_x reduces further as λ_x decreases, justifying the neglect of compressional–mode waves at small horizontal scales.

Under these conditions, Maxwell's equations are reduced to two equations in $\tilde{E}_x$ and $\tilde{B}_y$:

$$\frac{\partial \tilde{E}_x}{\partial z} = -\left(\frac{k_x^2}{\mu_0\tilde{\sigma}_0 + i\omega/c^2} + i\omega\right)\tilde{B}_y \quad (5)$$

$$\frac{\partial \tilde{B}_y}{\partial z} = -\left(\mu_0\tilde{\sigma}_1 + \frac{i\omega}{c^2}\right)\tilde{E}_x \quad (6)$$

Ionospheric reflection characteristics are determined as follows. A shear Alfvén wave is "launched" from above the ionosphere (using an altitude of 1000 km). The dispersion relation for this wave is

$$\tilde{k}_{z,s} = \sqrt{-\left(\frac{k_x^2}{\mu_0\tilde{\sigma}_0 + i\omega/c^2} + i\omega\right)(\mu_0\tilde{\sigma}_1 + i\omega/c^2)} \quad (7)$$

(Note that the term "$i\omega$" above was erroneously omitted from Equation (13) of *Knudsen et al.* [1992].)

The wave propagates through the ionosphere to the perfectly reflecting surface on the ground and is reflected upwards. The reflection coefficient is calculated as the ratio of the reflected wave amplitude (at 1000 km altitude) to the incident wave amplitude.

The dependence of the reflection coefficient on physical parameters can be seen more clearly under the following approximation, the suggestion for which we thank J. Johnson.

In the low-frequency limit, the dispersion relation simplifies to:

$$\lambda_z \sim i\sqrt{\frac{\sigma_0}{\sigma_1}}\lambda_x \sim i\sqrt{\frac{m_i}{m_e}\frac{\nu_i^2 + \Omega_i^2}{\nu_i\nu_e}}\lambda_x \quad (8)$$

For typical parameters in the E region (m_i/m_e = 30, $\nu_e/\nu_i = 10$), where ions become demagnetized, the parallel damping length is thus $l_z \approx 10 - 20\lambda_x$. This corresponds to absorption of 99% of the wave, over a nominal E-region thickness of 40 km, for λ_x between 3 and 5 km, roughly consistent with the numerical results.

Note that the above Equation predicts that ionospheric reflection should depend strongly on ν_e, whereas *Knudsen et al.* [1992] showed very little dependence on ν_e in the limit of large λ_x. Field-aligned current j_z increases with k_x, leading to larger dissipation ($j_z E_z$) and non-negligible dependence on ν_e (via σ_0) in the small–λ_x limit.

3. Ionospheric parameters

As in *Knudsen et al.* [1992], a vertical background magnetic field that falls off as $(r/R_E)^{-3}$ is assumed. For each of the cases studied here, a Jacchia [*Jacchia*, 1971] neutral density model is used as a basis for calculating ion–neutral collision frequencies as described in *Schunk and Walker* [1973]. Electron–neutral and electron–ion collision rates are calculated as described in *Banks and*

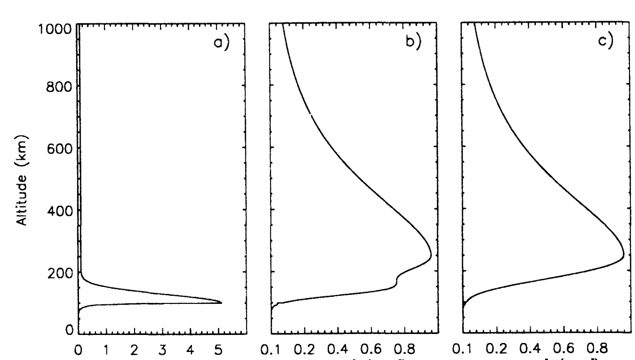

Figure 1. The density profiles studied in this work. The "E" profile (a) represents night–time periods of hard precipitation. The "EF" profile (b) represents a post–sunset region with auroral precipitation or a sunlit ionosphere and the "F" profile (c) represents a post–sunset period with no auroral particle precipitation.

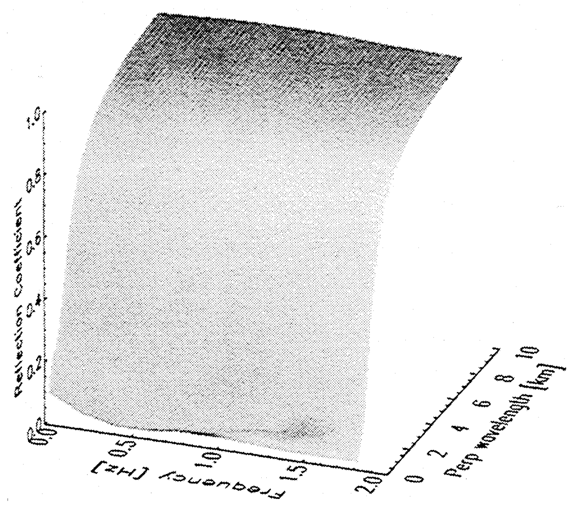

Figure 2. Calculated electric field reflection coefficients associated with an "E" profile for frequencies ranging from 0.2 to 2.0 Hz and λ_x from 0.1 to 10.0 km. Note the weak reflection for waves with perpendicular scale lengths less than $\sim 1-2$ km.

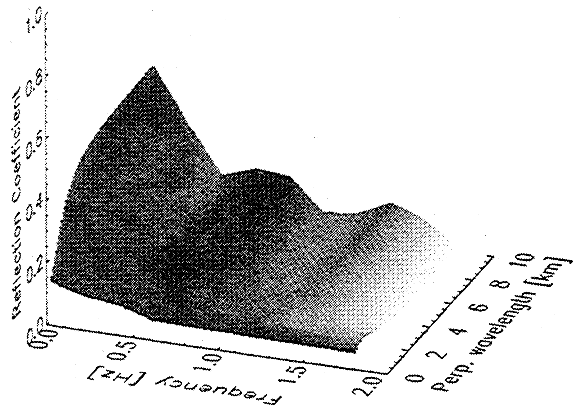

Figure 4. Same format as for Figure 2, but showing reflection coefficients for an "F" profile. Reflection is significant only for waves with frequencies below 0.4 Hz and λ_x above $\sim$ 2 km; reflection of waves with λ_x less than $\sim$ 2 km is poor.

Kockarts [1973] and then are summed to yield a net electron collision frequency.

In this paper, we present results from three different ionospheric density profiles, although we point out that virtually any realistic vertical profile can be studied with this model. The profiles considered here are the same as those examined by *Knudsen et al.* [1992] and are presented in Figure 1. Collision frequencies and conductivities are shown in that paper and are not repeated here.

The first profile, shown in Figure 1(a), is denoted "E", and is intended to be representative of night-time periods of hard precipitation, typical of what might be expected as a result of auroral precipitation.

Electric field reflection coefficients associated with our "E" density profile are presented in Figure 2. Note

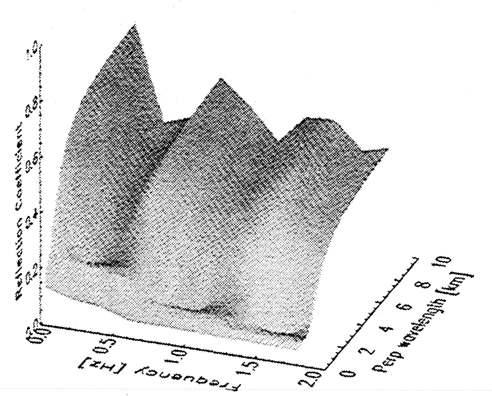

Figure 3. Same format as for Figure 2, but showing reflection coefficients for an "EF" profile. Reflection is poor for small perpendicular wavelengths, although a nominal value of $\sim$ 0.5 for perpendicular wavelengths longer than $\sim$ 2 km is a good rule of thumb.

the small dependence on frequency and, especially, the drop to near zero for short perpendicular wavelengths. To be strictly correct, compressional waves should be included in our calculations, but doing so results in stability problems for our model as perpendicular wavelengths become small. Since the objective here is precisely to understand the behavior of short wavelengths, compressional waves are omitted from our model, an omission which is justified because, for short perpendicular scales, the compressional mode is evanescent in z over distances comparable to λ_x.

In the next case studied, which we call "EF", a second density peak was added to the ionosphere, with the intention that the profile be typical of a post–sunset region with auroral precipitation or a sunlit ionosphere. The profile is presented in Figure 1(b) and electric field reflection coefficients for this profile are presented in Figure 3. Values are lower than those for an E region alone but also show frequency–dependent absorption. More importantly, reflections again drop to near zero for all frequencies with small perpendicular wavelengths.

Finally, we consider an "F" profile alone, intended to be typical of a post–sunset period with no auroral particle precipitation (see Figure 1(c)). Reflection coefficients for an "F" profile are presented in Figure 4 and are clearly lower than those for an "EF" profile. Again, reflections drop to near zero for small perpendicular wavelengths.

4. Conclusions

Our calculations yield the following results:

1. For all frequencies and all ionospheric profiles considered, reflection of shear Alfvén waves with perpendicular wavelengths of the order of $\sim$ 1 km or less is negligible. The model shows that the highest electric field reflection coefficient is associated with the "E" profile, although the coefficient is only $\sim$ .1, increasing to $\sim$ .5

for a wave with a perpendicular wavelength of 2 km. This result is consistent with *Borovsky* [1993], who estimated that sub-kilometer structures would not reflect from the ionosphere.

2. Reflection from an "F" profile is significant only for waves with frequencies below 0.4 Hz and λ_x above $\sim$ 2 km; reflection of waves with λ_x less than $\sim$ 2 km is poor. Models of cavity and waveguide mode pulsations often assume a perfectly reflecting ionosphere. The results presented here support the idea that ionospheric dissipation may be important in determining the lifetime of the field line resonances associated with these models.

3. These results imply that parallel electric fields associated with inertial Alfvén waves (which would be enhanced upon reflection from the ionosphere) would have limited magnitude for small-scale perpendicular wavelengths. One implication is that wave growth resulting from the Alfvén resonator [*Lysak*, 1999] may be limited when perpendicular scales are small.

4. Given that Alfvén waves with perpendicular scales of 1 km or less and frequencies below 2 Hz do not reflect from the ionosphere, any such waves that are propagating upward must have their source in the topside ionosphere. We note that thicknesses of the narrowest features within the optical aurora coincide with the sub-km scales shown here to be absorbed in the ionosphere, and can in fact be as small as tens of meters [*Borovsky*, 1993]. In principle, the ionospheric conductivity variations resulting from such structured electron precipitation could act as a source of upward-propagating small-scale Alfvén waves.

5. Finally, we note that field-line resonance (FLR) theories of auroral arcs rely on ionospheric reflection to generate auroral structure with horizontal scales as large as tens of kilometers and as small as 1 km or less (e.g. *Lotko et al.* [1998]; *Rankin et al.* [1999]). We have argued that smaller-scale structures do not reflect and therefore cannot be enhanced via field-line resonance. However, observations show that FLR theories reproduce many observed features of auroral arcs, at both large and small scales. This apparent contradiction implies either that the agreement between FLR models and observed small-scale structure is coincidental, or that the reflection model discussed in this paper omits significant physics in some situations. One notable omission is that of horizontal variations in ionospheric conductivity, which can affect wave reflection properties. This remains an important topic for further study.

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